ENERGY DISSIPATION IN AN UNDERGROUND EXPLOSION

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UDC 622.235.5

The dissipation of the energy of an explosion in soil has been discussed by Sagomonyan [1, 2] and by the author [3]. The dissipation processes occurring in the expansion of a cavity in an underground explosion are discussed in [1]. The temperature distribution in the earth directly behind the spreading shock wave is found in [2] for the two-dimensional case using the "plastic gas" model. Experiments to determine the temperature distribution in an underground explosion are described in [3]. The experimental data are in good agreement with theoretical predictions made by solving the nonstationary heat diffusion problem using a mathematical formulation corresponding closely to the experiment. In the present article we analyze the initial temperature distribution after motion ceases. Ratios are found of the fraction of the total energy of the explosives expended in heating the ground in an irreversible compression by the shock front, the fraction dissipated in plastic flow behind the front, and the fraction remaining in the detonation products after the edge of the cavity stops moving.

The most complete model of soft earth was proposed by Grigoryan,* but this model is very involved for analytic calculations. Since experiment shows that the major portion of the energy of an explosion is dissipated in a layer of earth lying close to the edge of the cavity, we use a simpler model of the motion.

We solve the problem within the framework of the model proposed by A. S. Kompaneets [4], especially since the equation of state for soil used in [4] corresponds rather closely to the experimental curve characteristic of loams if it is assumed that "packing" begins at pressures ~5-10 kg/cm^2.

For spherical symmetry we have, according to [4], the following formulation of the problem:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)=\frac{\partial\sigma_r}{\partial r}+\frac{2(\sigma_r-\sigma_\theta)}{r},\quad \frac{\partial}{\partial r}(r^2U)=0 \tag{1}$$

with the plasticity condition

$$\sigma_r-\sigma_\theta=k+m(\sigma_r+2\sigma_\theta) \tag{2}$$

and the following conditions at the edge of the cavity and at the shock front:

$$\sigma_r(a)=-P(a)$$
$$\sigma_r(R)=-\rho_0\xi\dot{R}^2-P_* \tag{3}$$

TABLE 1

ξ	m	a_k/a_0	E_1, %	E_2, %	E_3, %
	0.1	9.58	46.7	44.6	8.7
0.05	0.233	7.7	31.5	58.1	10.4
	0.5	6.15	21.6	65.9	12.5
	0.7	5.6	18.2	68.4	13.4
	0.1	9.44	55.7	35.5	8.8
0.1	0.233	7.97	41.2	48.7	10.1
	0.5	6.67	30.4	57.8	11.8
	0.7	6.21	26.6	61.1	12.3

Here R is the radius of the shock front, a is the radius of the cavity, σ_r, $\sigma_\theta=\sigma_\varphi$ are the components of the stress tensor, r is the radial coordinate, u is the mass velocity of the particles, P(a) is the pressure in the cavity, and P_* is the pressure at the start of the irreversible compression.

The relation between the radius of the cavity and the radius of the shock front is found from the law of conservation of mass:

$$a=[\xi+(1-\xi)(a_0/R)^3]^{1/3}R=\varepsilon R \tag{4}$$

* S. S. Grigoryan, Studies in the Mechanics of Soils [in Russian], Doctoral Dissertation, Moscow State University (1965).

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 184-187, September-October, 1972. Original article submitted February 21, 1972.

where $\xi = 1 - \rho_0/\rho$ is the packing.

By using (2)-(4) we obtain from Eq. (1) the equation for the coordinate of the front R:

$$R\ddot{R} + A(R)\dot{R}^2 = B(R)[P(a) + C(R)] \tag{5}$$

Here

$$A(R) = 2 - \frac{(1-\xi)(\alpha-1)}{\varepsilon^{\alpha-1}-1} - \frac{2\xi(\alpha-1)(1-\varepsilon^{\alpha-4})}{(\alpha-4)(1-\varepsilon^{\alpha-1})}$$

$$B(R) = \frac{(\alpha-1)\varepsilon^{\alpha}}{\rho\varepsilon(1-\varepsilon^{\alpha-1})}, \quad C(R) = \frac{k}{3m}(\varepsilon^{-\alpha}-1) - P_*\varepsilon^{-\alpha}, \quad \alpha = \frac{6m}{1+2m}$$

Dots over letters denote time derivatives.

In Eq. (5) the function P(a) is defined by the relations [5]

$$P(a) = \begin{cases} P_0(a/a_0)^{-3\gamma_1}, & \text{if} \quad a_0 \leqslant a \leqslant a_* \\ P_0(a_*/a_0)^{-3\gamma_1}(a/a_*)^{-3\gamma_2}, & \text{if} \quad a_* < a \end{cases}$$

$$a_* = 1.53\, a_0, \quad \gamma_1 = 3, \quad \gamma_2 = 1.27$$

We reduce Eq. (5) to dimensionless form:

$$x\frac{dy}{dx} + 2Ay = 2B(\beta x^{-3\gamma}\varepsilon^{-3\gamma} + C)$$

$$x = R/R_0, \quad y = \dot{R}^2\xi\rho_0/P_0$$

Here

$$\beta = \begin{cases} 1, & \text{if} \quad a_0 \leqslant a \leqslant a_* \\ (a_0/a_*)^{3(\gamma_1-\gamma_2)}, & \text{if} \quad a_* < a \end{cases}$$

$R_0 = a_0$ is the initial radius of the cavity, and ρ_0 is the initial density of the soil.

This equation was integrated on a computer for $P_0 = 7.97 \times 10^4$ kg/cm², $P_* = 6$ kg/cm², and k = 1.41 kg/cm². Figure 1 shows y as a function of x for various values of ξ and m. The number 1 in Fig. 1 denotes curves corresponding to m = 0.7, $\xi = 0.05$ (solid curve) and m = 0.7, $\xi = 0.1$ (open curve); the number 2 denotes curves for m = 0.1, $\xi = 0.05$ (solid curve) and m = 0.1, $\xi = 0.1$ (open curve). The curves are not extended beyond x = 5 since for the scale chosen they can hardly be distinguished and merge into a single curve. Table 1 shows the values of the final radius of the cavity for various values of ξ and m.

The expression for the energy of the shock compression per unit mass has the form

$$e_1 = \frac{1}{2}\left(\frac{1}{\rho_0} - \frac{1}{\rho}\right)(P + P_*) = \frac{\xi P_0}{2\rho_0}\left(y + \frac{2P_*}{P_0}\right) \tag{6}$$

The change in the energy of plastic deformation per unit mass is

$$\frac{de_2}{dt} = \frac{2u}{\rho r}(\sigma_\theta - \sigma_r)$$

By using the plasticity condition we write this expression in the form

$$e_2 = -\frac{\alpha}{\rho}\int_{t_0}^{t_k}\left(\sigma_r + \frac{k}{3m}\right)\frac{u}{r}\,dt \tag{7}$$

Here t_0 is the time the shock wave reaches the point r_0, and t_k is the time the whole motion ceases.

From the solution of the equations of motion we have for σ_r

$$\sigma_r + \frac{k}{3m} = \frac{\rho\xi}{\alpha-1}(2\dot{R}^2 + R\ddot{R})\left[\frac{R}{r} - \left(\frac{R}{r}\right)^{\alpha}\right] -$$
$$- \frac{2\rho\xi^2}{\alpha-4}\dot{R}^2\left[\left(\frac{R}{r}\right)^4 - \left(\frac{R}{r}\right)^{\alpha}\right] - \rho_0\xi\dot{R}^2\left(\frac{R}{r}\right)^{\alpha} + \left(\frac{k}{3m} - P_*\right)\left(\frac{R}{r}\right)^{\alpha} \tag{8}$$

From the law of conservation of mass it is easy to obtain a relation between R and r, and U and $\dot{R}$:

$$r = [\xi R^3 + (1-\xi)r_0^3]^{1/3}, \quad U = \xi(R/r)^2\dot{R} \tag{9}$$

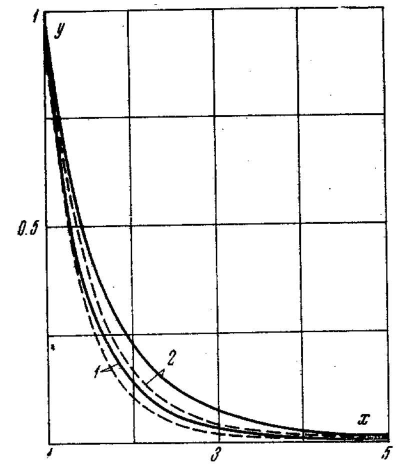

Fig. 1

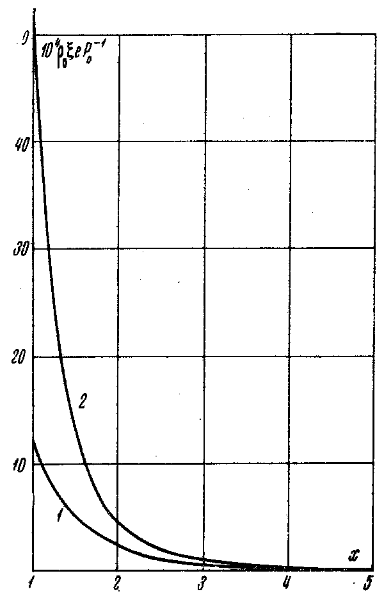

Fig. 2

By substituting (8) and (9) into (7) and changing to R as an integration variable by the relation $dR = \dot{R} \cdot dt$ we have after transformations

$$e_2 = \frac{6\alpha\xi^3}{(\alpha-4)(\alpha-1)} \int_{r_0}^{R_k} \left(\frac{R}{r}\right)^7 \dot{R}^2 \frac{dR}{R} + \frac{d\xi^3(3+\alpha)}{2(\alpha-1)} \int_{r_0}^{R_k} \left(\frac{R}{r}\right)^{6+\alpha} \dot{R}^2 \frac{dR}{R} - \frac{\alpha\xi^2(2\alpha\xi-\alpha-4\xi+4)}{2(\alpha-4)} \int_{r_0}^{R_k} \left(\frac{R}{r}\right)^{3+\alpha} \dot{R}^2 \frac{dR}{R} - \frac{\alpha\xi}{\rho}\left(\frac{k}{3m} - P_*\right) \int_{r_0}^{R_k} \left(\frac{R}{r}\right)^{\alpha+3} \frac{dR}{R} \qquad (10)$$

Equation (10) is easily put into dimensionless form by multiplying it by $\rho_0\xi P_0^{-1}$. All the integrals in (10) were evaluated by computer. Figure 2 shows the dimensionless quantities e_1 and e_2 as functions of x for $\xi = 0.05$ and $m = 0.233$. The numbers 1 and 2 denote, respectively, e_1 and e_2 as functions of x. The curves indicate that a large part of the energy of the explosion is dissipated within a distance $\sim 5\ R_0$. After all motion ceases this layer of earth has a thickness $\sim(0.05\text{–}0.15)\ a_k$, where a_k is the final radius of the cavity.

The total fractions of the energy of the explosion going into shock compression and plastic flow are given by

$$E_1 = \int_{R_0}^{R_k} 4\pi e_1 \rho_0 r^2 dr, \qquad E_2 = \int_{R_0}^{R_k} 4\pi e_2 \rho r^2 dr$$

A fraction E_3 of the total energy of the explosion is contained in the detonation products. The quantitative relations between the various fractions of the energy of the explosion for various characteristics of soils are shown in Table 1.

It is clear from the data presented that a large part of the energy of the explosion is accounted for by plastic flow behind the shock front, and not taking account of this part of the energy of the explosion can lead to a large error in determining the initial temperature distribution in the explosion. The conclusion is that the initial temperature distribution in the earth after the explosion has a delta function form with the maximum temperature at the edge of the cavity and a very rapid decrease with distance.

LITERATURE CITED

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